Abstract

In a first experiment, subjects performed a variant of the same-different tasks, here designed as pattern comparison and categorization with different alternatives (categories) necessary to select. We found that the entropy reduction depended (among other things) on the number of alternatives and increased with an increased number. In a second experiment, subjects had to solve two kinds of mathematical problems with different alternatives (strategies) necessary to select. We found that the entropy reduction increased with the number of alternatives too. In spite of the large response time difference between these two experiments the entropy reduction is caused by the stability in time of the microstates - measured by means of the EEG-coherence - both in memory and thinking experiments.

Problem:

Everybody is convinced that the similarity between the external and the internal psychophysical functions corroborates the theoretical approach in psychophysics. Exploring the cortical evidence of perception Romo (2001) restated this similarity in acoustical threshold experiments with monkeys. In memory psychophysics more complex tasks are investigated, e.g. categorization and problem solving based on cognitive structures. The longer response time or response time difference alone, specially in problem solving does not allow the detailed analysis of the cognitive process. This complex psychophysical processes need other approaches. One of this approach concerns the old idea by Potts (1975) in his famous article: Bringing order to cognitive structures (see also Restle, 1975). We start from this idea and try to measure the internal ordering process to accompanied the external observable relationship between an task and the behavioural data by means of the internal entropy reduction in terms of the internal psychophysics.

Definition of entropy reduction:

Theoretically, according to Boltzmann (1872), the entropy increases with the number of alternatives to classify elements into classes. These are prototypical tasks in human memory and thinking, if we are able to substitute the elements by internal elements as concepts or strategies. Obviously the entropy increases with the increase of the number of alternatives. Unfortunately we are unable to measure this theoretical entropy respectively the entropy reduction because of the instability of the elements (concepts, strategies, features etc.). Instead of that, Shannon’s entropy with regard to the distribution of the different states gives evidence of the disorder of the occurrence of the states. The states represent internal activities whereas the relationship between the elements and the states are unknown for the time being. Nevertheless the states might be interpreted. The hidden order may be described by transition probabilities and quantified by the conditional entropy of the occurrence of a state observing the preceded state. The difference between Shannon’s entropy and the conditional entropy is denoted by entropy reduction H_{red} and reflects the sequential structure of the states or the cutback of disorder.

* Sponsored by the DFG (Scha 741/1-4).
The entropy reduction is

\[ H_{\text{red}} = H - \sum_{i} P(i) \cdot H(i) \quad , \quad i = 1, \ldots, n_p \]  

with Shannon’s entropy

\[ H = -\sum_{j} P(j) \cdot \log(P(j)) \quad , \quad j = 1, \ldots, n_p \]  

and the conditional entropy

\[ H(i) = -\sum_{j|i} P(j|i) \cdot \log(P(j|i)) \quad , \quad i, j = 1, \ldots, n_p \]  

where \( i, j \) are the numbers of clusters and stand for the different microstates and \( \log \) denotes the logarithm dualis.

**Measurement of entropy reduction:** The investigated measure of entropy reduction \( H_{\text{red}} \) is observed by constructing chains of EEG synchronization states. For each task individual alphabets of 6 prototypes of synchronization states were determined by means of instantaneous EEG coherence (Schack, et al., 1999, 2001). The strength of concatenation of the microstates may be expressed by this special entropy measure. This sequential property of microstates is the dependent variable and reflects the cognitive process in terms of internal psychophysics. This is the new idea. It is quite obvious that this neuroscience approach does not allow the identification of elements mentioned above. Nevertheless the microstates can be interpreted. Against this background we expect that at least one of this 6 microstates can be interpreted in terms of executive control, necessary to decide between alternatives.

The EEG was recorded from 19 scalp electrodes (10/20 system, ear lob reference, 256 Hz). Instantaneous coherence analysis (see e.g. Schack et al. 1999) was performed for 30 electrode pairs. The 30-dimensional vector of time courses of band coherences (13-18 Hz) for each single trial was subdivided into segments with stable coherence values. Afterwards, the segments were clustered into six classes. The correspondent cluster centres compose the individual alphabet of states of synchronous oscillatory activity (see e.g. Krause et al. 2000). This states are denoted as microstates. The procedure of data analysis is shown in Fig. 1.

**Entropy reduction in memory:**

**Experiment:** Three subjects (age 19, 3m) performed a variant of the same-different tasks, here designed as pattern comparison task and a categorization task using words and pictures as stimuli (see Fig. 2, only the word presentation is shown). Four categories (animals, trees, furniture and clothes) with ten examples in each category were presented in order to avoid a stereotypical answer. The tasks and the categories were randomly mixed whereas the two kinds of word and picture presentation were given in blocks with 80 trials per block. All in all each subject performed 1600 trials. On each single trial, subjects observed two words or two pictures presented on the centre of a computer screen. Subjects started each trial by pressing a button and after a time delay of 300 ms the task question (the question “same?” for pattern comparison task, respectively the question “same category?” for categorization task), a first stimulus and after an inter-stimulus interval a second stimulus were given. Subjects responded to each stimulus presented by pressing one of two buttons on a response box held in the right hand. Responses were limited to “yes” and “no”. For pattern comparison, the subjects decided the identity of the two words or the two pictures. In case of categorization subjects were to classify the two words or the two pictures presented. In this way a category concept from four possible categories had to be select in contrast to pattern comparison, independent of the kind of stimulus presented.

**Result:** As shown in Fig. 2, response times differ significantly between tasks (categorization versus pattern comparison, word: \( t(2) = 7.132; P < 0.05 \); picture: \( t(2) = 8.375, P < 0.05 \)). Subjects need more response time with categorization than with pattern comparison. This well known effect (Posner and Mitchell, 1967) was found both for word and picture presentation. The entropy reductions differ significantly too between tasks (categorization versus pattern...
comparison, word: t(2) = 7.132; P < 0.05, picture: t(2) = 8.375, P < 0.05). As expected the entropy reduction is higher in case of a larger number of alternatives. The markovian chains demonstrate the reason of the increase of the entropy reduction. The nodes denotes the six microstates (ordered by means of the maximum self-transition probability) and the arrows the transition probabilities with significant differences between pattern comparison and categorization. Obviously in contrast to pattern comparison, categorization is characterized by a higher self-transition probability. The increase of entropy reduction is mainly caused by an increase of the self-transition probability of the microstates. A high self-transition probability hints to the stability in time of the correspondent microstate.

**Entropy reduction in thinking:**

**Experiment:** Twelve right handed mathematically highly gifted (age 18.3, 9m, 3f, IQ 124) subjects carried out two different classes of tasks with different task complexity (Fig. 3). The task complexity was quantified by the number of modality strategies selectable in order to solve the problem. The two kinds of tasks were given in blocks, presented on a computer screen.

**Result:** As shown in Fig. 3, response times differ significantly between the two classes of tasks with different alternatives (z=-3.06, p<0.002). The entropy reductions differ significantly too (z=-2.93, p<0.003). As expected the entropy reduction is higher in case of larger number of alternatives. The markovian chains explain the effect as mentioned above. In contrast to addition, the increase of entropy reduction in solving elementary mathematical problems is mainly the result of an increase of the self-transition probability of such microstates with an increased coherence at the electrode pairs F3/F7 and F3/Fz (z=-2.69, p<0.036). This support the assumption that a higher number of alternatives necessary to select is accompanied by a higher stability in time of the correspondent microstate, in human memory as well as in human thinking.

**Conclusion:**

The internal processes of memory and thinking tasks show a big similarity in respect of the stability in time of the microstates although the external processes show a big dissimilarity in respect of their response times.

**References:**


Fig. 1: illustration of segmentation and cluster procedure
pattern comparison and categorization

<table>
<thead>
<tr>
<th>tasks:</th>
<th>pattern comparison</th>
<th>categorization</th>
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<tbody>
<tr>
<td>alternatives:</td>
<td>low (1)</td>
<td>higher (4)</td>
</tr>
<tr>
<td>example:</td>
<td>same ?</td>
<td>same category ?</td>
</tr>
<tr>
<td></td>
<td>sheep</td>
<td>sheep hare</td>
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response time

entropy reduction:

markovian chain:

Fig. 2: Memory: word presentation. Response time (ms), entropy reduction and markovian chain for pattern comparison and categorization with word presentation. The arrows denote transition probabilities with significant differences between pattern comparison and categorization ($z = -1.606, p = 0.109$).
addition and elementary mathematical problem

tasks: addition elementary problem
alternatives: low (1) higher (2)
example: $5+3+9 = ?$ (computationally solvable).
The length of the diagonal $d$ of a square is 5 cm. How long is the length of an edge of a square with doubled area? (computationally and imaginably solvable).

![Graph showing response time and entropy reduction for addition and elementary problem tasks.

![Graph showing markovian chain for addition and elementary problem tasks.

Fig. 3: Thinking: addition and elementary mathematical problem. Response time (s), entropy reduction and markovian chain for addition and elementary mathematical problems. The arrows denote transition probabilities with significant differences between pattern comparison and categorization ($z = -2.134, p = 0.033$).